

Observed Property of σ -Meson and Chiral Symmetry

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The phenomenologically observed property of $\sigma(600)$ in our recent analyses of various $\pi\pi$ -scattering and -production processes is reviewed and compared with the prediction of $SU(2)$ linear σ model. Furthermore, a possibility of the scalar σ -nonet ($\sigma(600)$, $\kappa(900)$, $f_0(980)$ and $a_0(980)$) forming with the pseudoscalar π -nonet a linear representation of $SU(3)$ chiral symmetry is investigated. The origin of repulsive background phase shift δ_{BG} , which is essential to lead us to the σ -existence in our phase shift analysis, is shown to come from the repulsive $\lambda\phi^4$ interaction.

§1. Introduction

Recently we have found rather strong evidence for existence of the light σ -meson through a series of analyses of $\pi\pi$ -scattering and various $\pi\pi$ -production processes. First we collect the values of obtained mass and width of σ -meson. We also review our consideration, whether the σ -meson with this property deserves to be the member of a linear representation of chiral $SU(3)$ symmetry or not. Secondly we explain that the origin of repulsive background phase shift δ_{BG} , introduced in our phase shift analysis and led to σ -existence, is compensating $\lambda\phi^4$ interaction, which is necessary from the viewpoint of chiral symmetry. Finally comparison of our method of phase shift analysis with the other methods, which did or did not lead to σ -existence, is given in some detail.

§2. Observed Property of σ -Meson and Linear σ Model

Observed Properties of σ -Meson obtained through in our recent analyses of the various processes are given in Table I.

Table I. Observed mass and width of σ -meson in our analyses

Process	m_σ/MeV	Γ_σ/MeV
<u>Present Our Estimate</u>	<u>500~675</u>	<u>300~500</u>
$\pi\pi$ -scattering ¹⁾	535~675	385 ± 70
(pp central collision ²⁾)	590 ± 10	710 ± 30
$J/\psi \rightarrow \omega\pi\pi$ decay ²⁾	480 ± 5	325 ± 10
$p\bar{p} \rightarrow 3\pi$ ^{0 3)}	540^{+36}_{-29}	385^{+64}_{-80}
$\Upsilon^{(m)} \rightarrow \Upsilon^{(n)}\pi\pi, \psi \rightarrow J/\psi\pi\pi$ ⁴⁾	$531 \pm$	$281 \pm$

$SU(2)L\sigma M$ In the $SU(2)$ linear σ model the coupling constant $g_{\sigma\pi\pi}$ of the $\sigma\pi\pi$ interaction is related to the λ of the $\lambda\phi^4$ interaction and m_σ as

$$g_{\sigma\pi\pi} = f_\pi \lambda = (m_\sigma^2 - m_\pi^2)/(2f_\pi). \quad (2.1)$$

Thus, the width of σ -meson Γ_σ is related with its mass m_σ through the relation $\Gamma_{\sigma\pi\pi}^{\text{theor}} = \frac{3g_{\sigma\pi\pi}^2}{4\pi m_\sigma^2} p_1$, $p_1 = \sqrt{\frac{m_\sigma^2}{4} - m_\pi^2}$. This relation between m_σ and Γ_σ is not derived from the non-linear treatment of σ meson. By using this relation with our presently estimated value of m_σ in Table I we can predict the value of Γ_σ as $\Gamma_\sigma^{\text{theor}} = 300\text{MeV}$ (for $m_\sigma=500\text{MeV}$) $\sim 900\text{MeV}$ (for $m_\sigma=675\text{MeV}$), which is consistent⁵⁾ with our present experimental estimate of Γ_σ given in Table I. Thus, the observed σ meson may be identified with the σ meson described in the L σ M.

$SU(3)$ L σ M Through the analysis⁶⁾ of $K\pi$ -scattering phase shift by a similar method to $\pi\pi$, we have obtained an evidence of existence of the $I = 1/2$ scalar $\kappa(900)$ -meson.

Now we have the scalar mesons below 1 GeV, $\sigma(600)$, $\kappa(900)$, $a_0(980)$ and $f_0(980)$. Previously I discussed that the properties of these scalar mesons are consistent⁷⁾ with those predicted by $SU(3)$ L σ M, as shown in Table II.*

Table II. The properties of the scalar meson nonet predicted by $SU(3)$ L σ M, compared with experiments. The underlined values of m_σ and m_κ along with f_π , m_π , m_η and $m_{\eta'}$ are used as inputs, and we can predict the masses and widths of scalar mesons.

	$m^{\text{theor}}/\text{MeV}$	$m^{\text{exp}}/\text{MeV}$	$\Gamma^{\text{theor}}/\text{MeV}$	$\Gamma^{\text{exp}}/\text{MeV}$
σ	<u>535</u> \sim <u>650</u>	<u>535</u> \sim <u>650</u>	400 \sim 800	385 ± 70
κ	<u>905</u> $^{+65}_{-30}$	<u>905</u> $^{+65}_{-30}$	300 \sim 600	545 $^{+235}_{-110}$
$\delta = a_0(980)$	900 \sim 930	982.7 ± 2.0	110 \sim 170	95 ± 14
$\sigma' = f_0(980)$	1030 \sim 1200	993.2 ± 9.5	0 \sim 300	67.9 ± 9.4

Being based on these results, it may be plausible to regard $\sigma(600)$, $\kappa(900)$, $a_0(980)$, and $f_0(980)$ as members of the scalar nonet, forming with the members of π -nonet a linear representation of the $SU(3)$ chiral symmetry.** Here it is also to be noted that the octet members of this scalar nonet, κ , a_0 and σ_8 (mixture of σ and f_0), satisfies⁷⁾ the Gell-Mann Okubo mass formula.

§3. Origin of Repulsive Phase and Comparison with Other Analyses

Origin of the δ_{BG} An important physical reason, which led us to σ -existence, is introduction of the background phase δ_{BG} *** in our phase shift analysis. The origin of this δ_{BG} is considered to have a close connection to the $\lambda\phi^4$ -interaction in L σ M⁵⁾ as follows: The $I = 0$ $\pi\pi$ -scattering amplitude is given by $3A(s, t, u) + A(t, s, u) + A(u, t, s)$. The s -channel term, $3A(s, t, u)$, is main contribution, which is given by $SU(2)$ L σ M as

$$3A(s, t, u) = 3(-2g_{\sigma\pi\pi})^2/(m_\sigma^2 - s) - 6\lambda \equiv \mathcal{K}_\sigma + \mathcal{K}_\lambda, \quad (3.1)$$

* The predicted properties are very sensitive to the ratio of f_K/f_π . We prefer the region of this ratio $1.329 < f_K/f_\pi < 1.432$ (somewhat larger than the experimental value 1.22), where the value m_κ^{theor} reproduces the experimental value within its uncertainty.

** This interesting assignment was suggested and insisted upon repeatedly by M. D. Scadron.⁸⁾

*** This strong repulsive phase shift is cancelled out by the attractive contribution from δ_σ in $I = 0$ channel, and not observed directly. However, in $I = 2$ channel there is no known/expected resonance, and the repulsive phase shift is directly observed experimentally, and can be fitted¹⁾ well by the hard core formula $\delta^2 = -p_1 r_c$ with core radius $r_c = 0.87\text{GeV}^{-1}$ (0.17fm).

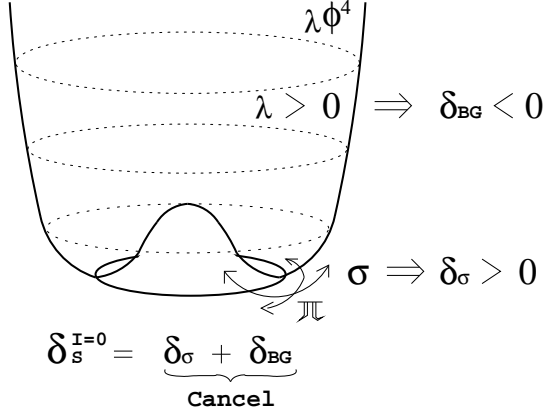


Fig. 1. “Mexican hat” potential, which may be considered as QCD effective potential. The repulsive interaction between pions due to $\lambda\phi^4$ term in $L\sigma M$ is the origin of negative phase δ_{BG} . Total phase shift δ is given by $\delta = \delta_\sigma + \delta_{BG}$ (δ_σ being due to σ formation).

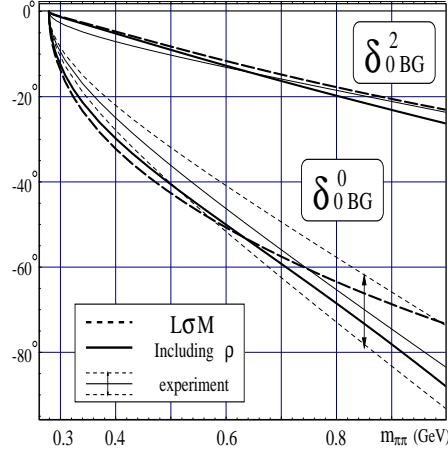


Fig. 2. The repulsive δ_{BG} in $I = 0$ and $I = 2$ $\pi\pi$ scattering predicted by the $L\sigma M$ with and without ρ -meson contribution. The results are compared with the phenomenological δ_{BG} of hard core type.

where we define, for later convenience, the 1st term as \mathcal{K}_σ and the 2nd term as \mathcal{K}_λ . Because of the relation (2·1), the dominant part due to σ -resonance (1st term) is cancelled by that due to repulsive $\lambda\phi^4$ interaction (2nd term) in $O(p^0)$ level:

$$3A(s, t, u) = \frac{3}{f_\pi^2} \frac{(m_\sigma^2 - m_\pi^2)^2}{m_\sigma^2 - s} - \frac{3(m_\sigma^2 - m_\pi^2)}{f_\pi^2} = 3 \frac{s - m_\pi^2}{f_\pi^2} + \frac{3}{f_\pi^2} \frac{(m_\pi^2 - s)^2}{m_\sigma^2 - s}, \quad (3.2)$$

where in the last side the $O(p^2)$ Tomozawa-Weinberg amplitude and the higher order correction term are left. As a result the derivative coupling property of π as Nambu-Goldstone boson is preserved. This strong cancellation in $L\sigma M$ corresponds to that between δ_σ and δ_{BG} in our phase shift analysis. Thus the origin of δ_{BG} has been proved to be the “compensating repulsive interaction” necessarily required from the viewpoint of chiral symmetry,* as schematically shown in Fig. 1.

In Fig. 2 I have shown our theoretical δ_{BG} estimated semi-quantitatively and compared with the phenomenological δ_{BG} of hard core type. We have unitarized the Born amplitude \mathcal{K}_σ and $\mathcal{K}_{BG}(= \mathcal{K}_\lambda$ in Eq.(3·1)), separately, and identified the obtained phases with δ_σ and δ_{BG} , respectively. In actual calculation in addition to \mathcal{K}_λ we also included the t, u -channel amplitudes $A(t, s, u)$, $A(u, t, s)$ and the ρ -meson contribution** in the \mathcal{K}_{BG} , and unitarized it following the N/D method. A subtraction constant necessary to make the D convergent was fixed so as to give $\delta_\sigma^{\text{th}} = \delta_\sigma^{\text{phys}}$ at $s = m_\sigma^2$. Our predicted δ_{BG} are seen to be almost consistent¹⁰⁾ with the phenomenological δ_{BG} in $I=0$ and $I=2$ channels obtained in our phase shift analysis.

* It should be noted that in the analysis⁹⁾ by Kaminski et al., leading to the light σ -existence, this repulsive phase shift is implicitly introduced through the use of the Yamaguchi-type potential.

** ρ -meson contribution is included by using the Schwinger-Weinberg Lagrangian.

Relation to the other phase shift analyses In our method the \mathcal{K}_σ and \mathcal{K}_{BG} (mainly coming from \mathcal{K}_λ) are unitarized separately, and the total S -matrix is given by

$$S \equiv \frac{1 + i\rho_1 \mathcal{K}}{1 - i\rho_1 \mathcal{K}} = S_\sigma S_\lambda; \quad S_\alpha = \frac{1 + i\rho_1 \mathcal{K}_\alpha}{1 - i\rho_1 \mathcal{K}_\alpha}, \quad \alpha = \sigma, \lambda, \quad (3.3)$$

where the simple \mathcal{K} -matrix unitarization method is applied to \mathcal{K}_λ , as well as \mathcal{K}_σ . The total \mathcal{K} -matrix defined above is given by

$$\mathcal{K} = \frac{\mathcal{K}_\sigma + \mathcal{K}_\lambda}{1 - \rho_1^2 \mathcal{K}_\sigma \mathcal{K}_\lambda} = \frac{(s - m_\pi^2) \frac{3(m_\sigma^2 - m_\pi^2)}{f_\pi^2}}{m_\sigma^2 + (1 - \frac{4m_\pi^2}{s}) \frac{9(m_\sigma^2 - m_\pi^2)^3}{(32\pi f_\pi^2)^2} - s} \approx \frac{(s - m_\pi^2) \frac{3m_\sigma^2}{f_\pi^2}}{m^2 - s}, \quad (3.4)$$

where Adler zero factor $(s - m_\pi^2)$ appears. The resulting S -matrix has a pole at the light mass $\approx m_\sigma$. The \mathcal{K} -matrix pole position is $s = m^2 \approx m_\sigma^2 + (\frac{3m_\sigma^3}{32\pi f_\pi^2})^2$, where the phase passes through 90 degrees. In case of $m_\sigma \approx 600$ MeV, m becomes ≈ 900 MeV. In this case, the S -matrix has a complex pole with mass $m_\sigma \approx 600$ MeV, but the phase passes 90° at $m \approx 900$ MeV. This situation of our phase shift analysis is common to the many other analyses motivated by L σ M. When the real Born amplitude, having one pole with Adler zero factor as in Eq. (3.4),

$$\mathcal{K} = (s - m_\pi^2)g^2/(m^2 - s), \quad (3.5)$$

is unitarized by \mathcal{K} -matrix or other methods, the light σ -meson pole in S -matrix is obtained generally:

In the analysis by Achasov et al.¹¹⁾ they obtain large $m \approx 1000$ MeV through the fit to the experimental data, but their physical mass is very small as $m_\sigma = 417$ MeV.

In the analysis by Tornqvist et al.,¹²⁾ their “Breit-Wigner” mass m_{BW} , which corresponds to m in our interpretation, is about 900 MeV, but the real part of the physical pole is $m_\sigma \approx 500$ MeV.

In the analysis by Igi and Hikasa,¹³⁾ the N/D -unitarization method is used. They use the large bare mass $m = m_\rho = 770$ MeV.*

In the analysis by Au, Morgan and Pennington,¹⁴⁾ the two-channel \mathcal{K} -matrix of the similar structure as Eq.(3.5)** with a polynomial background \mathcal{K}_{pol} was used. They obtained $m \approx 900$ MeV, however, they only searched for the case of small g -value, which corresponds to narrow resonance like $f_0(980)$, and overlooked the possibility of wide resonance with large g . However, σ is actually required to have wide width from Eq.(2.1). Accordingly they were forced to introduce the large

* They include the crossed channel ρ - as well as σ - meson exchange. They compare the phase shifts with and without σ -effect, and obtain the conclusion of σ -existence. It should be noted that their m_σ corresponds to our m . This mass is smaller than the other analyses motivated by L σ M, and thus the predicted phase shift passes through 90 degrees with somewhat smaller energy than the others. The real part of the S -matrix pole position, denoted as m_σ in our notation, is not given in their paper.

** They also try three pole fit, but the other two poles are far from the relevant energy region, and this analysis is essentially equivalent to the above one-pole fit.

background contribution \mathcal{K}_{pol} in order to fit experimental data^{***} Their result of no light σ -existence is simply to be due to this overlooking and is not correct.

§4. Concluding remark

The observed values of mass and width of scalar σ -nonet ($\sigma(600)$, $\kappa(900)$, $a_0(980)$ and $f_0(980)$) are consistent with the relation predicted by $SU(2)$ and $SU(3)$ $L\sigma M$. This fact suggests the linear representation of chiral symmetry is realized in nature. The strong cancellation between δ_σ due to the σ resonance and δ_{BG} introduced in our phase shift analysis corresponds to the cancellation between σ -resonance amplitude and repulsive $\lambda\phi^4$ amplitude in $L\sigma M$, and this cancellation is guaranteed from the viewpoint of chiral symmetry. The other phase shift analyses motivated by $L\sigma M$, leading to the σ -existence, also take into account this cancellation mechanism, explicitly or implicitly. The reason of missing σ in the phenomenological analysis by Au, Morgan and Pennington is that they overlooked the possibility of wide resonance, and did not pay attention to the cancellation mechanism of chiral symmetry breaking as depicted in Fig. 1, and thus their conclusion of no σ -existence is not correct.

Concerning on our analysis of $\delta^{I=0}$, Pennington made a criticism.¹⁵⁾ that the choice of δ_{BG} is completely arbitrary and accordingly “one can obtain more or less any set of Breit-Wigner parameters one likes for the σ .” However, The δ_{BG} has the clear physical origin in the compensating repulsive $\lambda\phi^4$ interaction, necessary from the viewpoint of chiral symmetry, and is not arbitrary. Energy dependence of δ_{BG} predicted by using $L\sigma M$ changes slightly depending on the unitarization methods, however, the pole position of σ is rather stable within the uncertainty given in our present estimate in Table I. Thus, the criticism is not valid.

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^{***} Furthermore, in their fit the effect of virtual $K\bar{K}$ -channel is extremely large in the energy region far below the $K\bar{K}$ -threshold. This fit seems to be unnatural, and necessarily to be corrected.